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The unit of mass—challenges in metrology

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Abstract : The SI unit of mass, the kilogram is the only one among the seven base units that now refers to a material artifact. Present state of our knowledge of the reproducibility and stability of the SI unit of mass and problems associated with this definition have been summarized. Perspectives for a new definition of the unit of mass based on fundamental physical constants or macroscopic quantum effects and problems associated with these possible definitions have been described. It has finally been concluded that the possible new definitions are not capable of being realized with accuracy greater than the one that it replaces and any successful replacement of the present unit of mass would require some rearrangement in the base units of the SI.

Keywords : SI mass unit, stability of mass standard, quantum mass standards

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1. Introduction

According to international system (SI) of unit of measurements, kilogram is the base unit of mass. The kilogram is the only SI base unit still defined in terms of a material artifact of platinum-iridium alloy, defined as follows

"The kilogram is the unit of mass : it is equal to the mass of the international prototype of the kilogram".

The international prototype of kilogram is a right circular cylinder whose height, 39 mm, is equal to its diameter. It is made of a single phase alloy of platinum-iridium with 10% iridium by weight. The alloy was made in London in 1879 by George Mathey in close collaboration with Sainte-Claire Deville and his assistant Jules Debray.

Accordingly, national prototypes are used to maintain the unit of mass in the member states of the Meter Convention. However, the problems associated with the present material artifact kilogram [1] may be summarised as follows :

(i) can be damaged or even destroyed,

(ii) not well-defined—accumulate foreign material and is difficult to clean reproducibly,

(iii) ages in an unknown manner—about 50 μg in the last 100 years (5×10^{-8}),

(iv) cannot be used routinely for fear of wear,

(v) available only in one laboratory.

2. Stability of the SI unit of mass

The international prototype kilogram and other platinum-iridium standard can be compared under the best experimental conditions with an uncertainty of 1 μg (1×10^{-9}) or even less.

It has been recently realized [2] that platinum-iridium prototype kilogram gains in weight according to the following relation :

$$\Delta m = 1\ \mu\text{g/year 'time'}.$$

This is to be understood as follows :

When national prototypes are brought to BIPM (International Bureau of Weights & Measures) for calibration against the working standards of BIPM they are weighed immediately after arrival. Then they are cleaned following the well-defined cleaning procedure, first in the alcohol and then in hot water steam. Then these are weighed again. Δm is the loss in mass after cleaning and 'time' is the time elapsed since the national prototype was last calibrated by BIPM. By implication, Δm is the gain in mass of the national prototype during the 'time'.

It is astonishing that this apparent gain in mass seems to go on for very long periods of time, atleast upto 50 years. Since a monolayer of absorbed moisture or air corresponds only to (1-2) μg if the surface is not greater than the macroscopic surface, it is difficult to imagine how so much material can be added in such a slow process. However, whatever is added to the mass must enter through the surface and it is therefore of interest to study possible micropoles and cracks of platinum surfaces.

The third periodic verification of national prototypes carried out by BIPM during 1989-92 [3] arrived at the following conclusions.

"The change in mass of prototypes fabricated in 1886 (numbers 1 to 40), provided they are stored and carefully used, is roughly +0.25 μg/year. The prototypes fabricated between the first and the second verification (numbers 44 to 55) showed a greater change in mass, about 0.9 μg/year. These changes are a combination of change in mass with respect to the international prototype and of wear occasioned by use. The possible causes of these changes are given below.

There is an increase due to surface contamination which is removed by cleaning, but some contamination like mercury, are not easily removed. There may be outgassing or diffusion to the inside depending on the concentration of gas with respect to equilibrium. Considering the fundamental impossibility of drawing definite conclusions from these results about the long term stability of the international prototype and its copies, the 20th CGPM (General Conference on Weights & Measures) recommended that national laboratories, pursue their work on related experiments, and develop new ones, with a view to monitoring the stability of the international prototype of the kilogram and in due course, opening the way to a new definition of the unit of mass based upon fundamental or atomic constants.

3. Quantum standards of mass

Nature has provided us various quantum effects involving fundamental constants which may be applied for the purpose of defining or maintaining the SI units. These have two advantages :

(1) they make use of phenomenon which are universally available,

(2) they often involve a parametric relationship with frequency.

The advantage of (1) is that because the phenomena are universal, one can in principle, avoid the necessity of having the traceability chain between the national laboratory and the ultimate users. The advantage of (2) is that once one knows the relationship between frequency and the parameter required, one may thereafter make the measurement entirely in terms of frequency which is the most accurately measurable quantity. Computations in atomic and molecular physics often use the Hartree system [4] in which the mass of the electron is the mass unit, the Bohr radius $\hbar/(m_e c\alpha)$ is the length unit and $\hbar/(m_e c^2\alpha^2)$ is the unit of time, where c is the velocity of light, α is the fine structure constant $\hbar = h/2\pi$, h being Plank's constant. The units used in the grand unification theories are called the Planck units. These are the units of mass, length and time given by;

the Planck mass $m_p = (\hbar c/G)^{1/2} = 2.17671\times10^{-8}$ kg,

the Planck length $l_p = (\hbar G/c^3)^{1/2} = 1.61605\times10^{-35}$ m

and the Planck time $t_p = (\hbar G/c^5)^{1/2} = 5.39056\times10^{-44}$ s,

where G is the universal gravitational constant.

Relations, similar to that of the Planck's mass, have also been obtained from other theories in physics [5] as we see in the following :

For a particle of inertial mass "m" obeying Dirac equation in quantum gravity, relations similar to that from Planck's mass are obtained from various theories in physics, as given below :

Weyl's Principle — General theory of relativity

$$Gm^2 = 2c\hbar,$$

Schwinger's Method — Quantum Electrodynamic

$$Gm^2 = 1/4(nc\hbar),$$

Coriolis field force — Particle physics

$$Gm^2 = 2c\hbar,$$

Mach Principle — Two body problem

$$Gm^2 = 2c\hbar,$$

Einstein Cosmological Relation

$$Gm^2 = (c\hbar)/2,$$

Line Element consideration in Quantum Gravitation Field

$$Gm^2 = (c\hbar)/2$$

Uncertainty Relation in four dimensions space time consideration

$$Gm^2 \geq (c\hbar)/2.$$

The above relations are same except the multiplier factor of $c\hbar$ on the r.h.s. Substituting the values of c, $\hbar$ and G as :

$$c = 299792458 \text{ m/s}, \quad h = 6.6260755\times10^{-34} \text{ J.s.},$$

$$G = 6.67259\times10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$$

in the relation $Gm^2 = c\,\hbar$ then $m = (c\hbar/G)^{1/2} = 10^{-5}$ g = 10^{-8} kg.

The fine structure constant α is given by :

$$\alpha = e^2/hc = 1/137,$$

where e is the charge of an electron, then in terms of fine structure constant

$$Gm^2 = 137e^2$$

or $$m = (137/G)^{1/2} e = 10^{-5}\ \mathrm{g} = 10^{-8}\ \mathrm{kg}$$

This is macroscopic quantity.

The term $\sqrt{G}m = (ch)^{1/2}$ is known as gravitational charge and is equal to $(\sqrt{137}\ e)$.

Thus, we can assume the existence of a fundamental particle of inertial mass 10^{-5} g and gravitational charge $\sqrt{137}\ e$. This particle has been given the name as UNITON or DYON in particle physics

To express mass m in terms of fundamental constants, G, c, h or α, e, G, we run into the following difficulties :

If we express $m = (ch/G)^{1/2} = e/(\alpha G)^{1/2}$,

	c is known within the accuracy of	0.004 ppm,
	e " " " " " "	0.3 ppm,
	h " " " " " "	0.6 ppm,
	α " " " " " "	0.045 ppm,
and	G " " " " " "	128 ppm.

Thus, to achieve an accuracy of even 1 ppm in measurement of m, we must have to know the value of G within the accuracy of 2 ppm. Therefore, it is desired to increase the accuracy of the measurement of G better than 2 ppm. Many experiments have been conducted by various scientists from time to time to measure the gravitational constant G more and more accurately. According to Long [6], the earlier data on the measurement of the gravitational constant suggest that the value of G is not constant at laboratory scale but it varies with the mass separation. The larger the separation, the larger is the value of G. This is another area where new experiments have to be carried out in search of the so called "fifth force", a short-range composition-dependent force postulated to exist at about the level of 1% of Newton's gravitational force.

A new definition of mass as the de Broglie frequency of an atomic particle leading to the new quantity $m'' = mc^2/h$ has been proposed [7]. But there are practical difficulties because one kilogram would correspond to an extremely large number, about 1.36×10^{50} per second. Alternatively, the kilogram could be defined on the basis of a neutron, but the realization of a mass of one kilogram would require the experimental determination of the neutron mass or another atomic mass in terms of the kilogram.

4. An optical definition of mass

According to Roy [8], an optical definition of mass is realizable in terms of length and time through the angular momentum properties of the photon measured using the torsion balance.

The photon energy $E = h\nu$ in its angular momentum form is given by $E = h\omega$. The unit of h is N.m.s^{-2} which is that of angular momentum

The constant $2\pi/h$ gives the circularly polarised photon flux required to produce a torque of 1 N.m from which mass may be derived in terms of the mechanical moment of inertia (I)

In absence of any added mass, the period of the torsion pendulum is given by

$$t_1 = 2\pi(I_1/\mu)^{1/2}, \tag{1}$$

where μ is the force constant.

If a mass m is added to the pendulum, the torsion period is given by

$$t_2 = 2\pi[(I_1 + I_2)/\mu]^{1/2}, \tag{2}$$

where the moment of inertia is given by

$$I = \sum mr^2. \tag{3}$$

From eqs. (1) and (2), I_1 can be eliminated. The force constant μ can be determined from the change in oscillation amplitude caused by the known applied optical torque

The added mass can be taken as an annulus of known dimensions, determined interferometrically. Since the dimensions of the annulus are accurately known, the mass of the annulus can be determined from the oscillation times

The fundamental limit to this measurement is the accuracy to which the Planck's constant is known. Currently, this is of the order of 10^6, giving a limit of 1 mg in 1 kg, for the torsion balance technique. Because the length is now fixed by the fraction of the distance travelled by light in one second, a similar approach may be used to define mass.

The flux of circularly polarized photons required to produce a torque of 1 N.m is $2\pi/h$, with a current value of $9.48251399\ (57) \times 10^{33}$ photons per second By fixing the value of $2\pi/h$, mass may be defined more accurately in terms of photon angular momentum.

Mass units based on fundamental physical constants mentioned above, are of atomic size while the artifact units of the SI are of macroscopic size. The involvement of the fundamental physical constants in our measurement system is not something that we should take for granted because it appears to violate a general principle of metrology which states that one tends to lose accuracy as one goes away from a unit.

Thus, we cannot measure a gram or a metric ton as accurately as a kilogram.

The 4th CCM (Consultative Committee for Mass and related quantities) meeting in 1991 [9] decided to prepare a report on a possible new definition of the kilogram. The report (CCM/93–10) prepared by M Glaser [10] summarized experiments that might establish a link either between the kilogram and electrical units or between the kilogram and fundamental constants such as the Avogadro constant.

5. Silicon Avogadro's constant

In SI, the relation between the atomic and macroscopic world is manifested in the definition of the Avogadro constant N_A, as the number of particles in one mole, *i.e.* the number of atoms in 0.012 kg of carbon 12.

As the Avogadro constant established an accurate relation between the unit of mass at the atomic scale and the present SI unit of mass, we can imagine an exact definition of Avogadro constant and an associated definition of the unit of mass based on a well-defined number of Si isotopes as follows :

One kilogram is the mass of 1/0.0289764969 of a mole of 28 Si.

This "special atomic" definition which would correspond to the definition of the second, would have a number of advantages over the present definition as following :

- It could, in principle, be realized everywhere.
- In practice, a single crystalline Si mass standard is likely to be without a number of the deficiencies that are presently associated with Pt–Ir mass standards.

But there are three problems with this definition of Mass Standard :

(1) Different samples of silicon seem to show different densities to a greater extent that would be expected by lattice spacing measurements.

(2) It still seem too difficult to fabricate an object of silicon whose volume can be determined with an accuracy of 1 part in 10^8 or better.

An accuracy of 1 part in 10^8 is the least that must be demanded of any atomic based definition of mass.

In recent measurements of the density of silicon, zerodur spheres were used as reference objects and their volumes were determined by interferometry [11]. The uncertainty in volume determination was about 5 parts in 10^7, made up of about equal parts of uncertainty in shape measurement and diameter measurements (interferometry).

Uncertainty in density measurement $= 5$ parts in 10^7,

Uncertainty in molar volume $= 2.4$ parts in 10^6.

The results obtained from the two different crystals of silicon show that the individual densities and atomic weights were within about 1.2 parts in 10^6, while the molar volumes were within 1.4 parts in 10^7 of each other. This confirms the hypothesis that

> "the crystals were indeed nearly perfect but they differed in isotopic composition".

(3) The correction for air buoyancy which at present is a limiting factor for the accuracy with which traceable mass measurements can be disseminated from primary mass standards of Pt–Ir to working standards of steel or brass, will be triple because of the low density of silicon.

These few results are enough to show that "atleast for the time being, any new definition of the unit of mass based upon silicon is far from being realizable".

6. Comparison of electrical and mechanical energies

This may be another method, appears to be much more promising which is based upon the comparison of electrical and mechanical energies

Using the classic current balance devised by Kibble *et al* at NPL, UK [12], it is feasible to carry out an experiment using a current balance in which a gravitational force Mg is compared with an electromagnetic force $I(d\phi/dy)$ This electromagnetic force is produced by means of an electric current I whose value is based upon the Josephson effect $(= 2e/h)$ and the quantum Hall effect $(= h/e^2)$.

The new idea of Kibble *et al* was to carry out a subsidiary experiment in which the coil was moved through the magnetic field while measuring at the same time the voltage produced and the velocity. Thus, there are two possible configurations of the experiment.

In the first configuration, $Mg = -I(d\phi/dy)$ and in the second we have

$$V = -(d\phi/dt) = -(d\phi/dy)\cdot(dy/dt) = -(d\phi/dy)\ u,$$

where $u = dy/dt$ is the velocity of the coil.

Hence, $Mg\cdot u = IV = V^2/R$ or $M = V^2/(R.g.u)$.

The acceleration due to gravity g can be measured within a few parts in 10^9 and both V and R can now be measured in terms of Josephson effect and quantum Hall effect within an accuracy of 1 part in 10^9.

Two experiments in this area are in progress. One at NPL, UK and the other at NIST in USA. At NPL, UK the new set up of the moving coil balance is enclosed in a vacuum chamber to avoid the buoyancy correction. A mass exchange mechanism is installed so that two mass standards of having different surface areas may be compared. The quoted reproducibility corresponds to one part in 10^7. A level of less than one part in 10^8 is expected in the future.

The NIST version of the Kibble [13] experiment is being carried out using superconducting coils. Instead of an equal-arm balance, a pulley is used. This takes the form of a rotating wheel over which is placed a wire. From this hangs, on one side, the moving coil, placed in the field of a superconducting magnet, and a pan for the one kg mass. The other side carries an auxiliary drive coil and a set of conventional electromagnets. The principle of measurement is identical with that used in the NPL experiment. The standard deviation obtained for a single measurement is 1.3×10^{-7} and is 2×10^{-8} for the mean of 48 measurements.

At this level of accuracy, it should be possible to combine the absolute measurements of watt with the known stability of a Josephson effect in order to monitor the stability of the mechanical units. Any instability in the mechanical units is most likely to be due to the lack of stability of the prototype kilogram. The long term limits of this method (a few parts in 10^9) are likely to be those imposed by the inconsistency of the acceleration due to gravity at the earth's surface and the difficulties of transferring weighings from vacuum to atmospheric pressure.

Once an accuracy of 10^{-8} or beyond is achieved, it will be possible to monitor the stability of the prototype kilogram by comparing electrical units against their corresponding mechanical units. At that stage, it will be necessary to consider how the prototype kilogram might be replaced. The answer will depend, critically, on how we wish to progress. For example, usually when a definition is replaced we expect both short-term and long-term gains in the accuracy with which the unit in question is realized.

It would not be much useful if the kilogram is replaced by electrical quantities as base unit. Also the gain in accuracy of the electrical units did not give a corresponding gain in accuracy of the realization of the mechanical units.

7. Conclusion

The present definition of the SI unit of mass, the kilogram, is often said to be unsatisfactory because it is based upon material artifact. Nevertheless, it is a very practical definition. The danger of "the kilogram" being lost due to damage or loss of the international prototype and its copies is not a very serious one. There are a sufficient number of national prototypes scattered around the world for a new international prototype to be reconstituted in the event of this happening.

The real problems with the present definition are that there is a little prospect of its being improved in its present form and that we have only very weak evidence as to its long term stability.

There is no way of defining the kilogram based upon fundamental constants or macroscopic quantum effects that would at present allow us to reach with about two orders of magnitudes of the short-term reproducibility in mass measurement, of a few parts in 10^9, available with the existing definition. The long-term stability of the kilogram is not worse than 5 parts in 10^9 per year. This is based upon the hypothesis that the international prototype and its official copies are not drifting in mass with respect to fundamental constants by more than ten times the rate at which they are drifting apart from one another (5 parts in 10^{10}) per year.

The possibility of replacing the kilogram by the quantum Hall effect and the Josephson effect might be a solution of the problem. However, to argue in favour of this type of solution, too far in advance, is to say that there are no further relevant quantum phenomenon to be discovered. Moreover, one would be arguing that mass metrology will not progress either and that is something that few are prepared to argue at this stage.

Any successful measurement of mass in terms of fundamental constants or macroscopic quantum effects would require some rearrangement in the base units of the SI. Such a change should certainly be included as the SI is made for men and not man for the SI.

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